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Rational Expressions - Multiply & Divide

Objective: Multiply and divide rational expressions.

Multiplying and dividing rational expressions is very similar to the process we use to multiply and divide fractions.

Example 1.

$\frac{15}{49} \cdot \frac{14}{45}$ First reduce common factors from numerator and denominator (15 and 7)

$\frac{1}{7} \cdot \frac{2}{3}$ Multiply numerators across and denominators across

$\frac{2}{21}$ Our Solution

The process is identical for division with the extra first step of multiplying by the reciprocal. When multiplying with rational expressions we follow the same process, first divide out common factors, then multiply straight across.

Example 2.

$\frac{25x^2}{9y^8} \cdot \frac{24y^4}{55x^7}$ Reduce coefficients by dividing out common factors (3 and 5)
Reduce, subtracting exponents, negative exponents in denominator

$\frac{5}{3y^4} \cdot \frac{8}{11x^5}$ Multiply across

$\frac{40}{33x^5y^4}$ Our Solution

Division is identical in process with the extra first step of multiplying by the reciprocal.

Example 3.

$\frac{a^4b^2}{a} \div \frac{b^4}{4}$ Multiply by the reciprocal

$\frac{a^4b^2}{a} \cdot \frac{4}{b^4}$ Subtract exponents on variables, negative exponents in denominator

$\frac{a^3}{1} \cdot \frac{4}{b^2}$ Multiply across

$\frac{4a^3}{b^2}$ Our Solution

Just as with reducing rational expressions, before we reduce a multiplication problem, it must be factored first.

Example 4.

$\frac{x^2-9}{x^2+x-20} \cdot \frac{x^2-8x+16}{3x+9}$ Factor each numerator and denominator

$\frac{(x+3)(x-3)}{(x-4)(x+5)} \cdot \frac{(x-4)(x-4)}{3(x+3)}$ Divide out common factors $(x+3)$ and $(x-4)$

$\frac{x-3}{x+5} \cdot \frac{x-4}{3}$ Multiply across

$$\frac{(x-3)(x-4)}{3(x+5)} \quad \text{Our Solution}$$

Again we follow the same pattern with division with the extra first step of multiplying by the reciprocal.

Example 5.

$$\frac{x^2-x-12}{x^2-2x-8} \div \frac{5x^2+15x}{x^2+x-2} \quad \text{Multiply by the reciprocal}$$

$$\frac{x^2-x-12}{x^2-2x-8} \cdot \frac{x^2+x-2}{5x^2+15x} \quad \text{Factor each numerator and denominator}$$

$$\frac{(x-4)(x+3)}{(x+2)(x-4)} \cdot \frac{(x+2)(x-1)}{5x(x+3)} \quad \text{Divide out common factors:}$$

$(x-4)$ and $(x+3)$ and $(x+2)$

$$\frac{1}{1} \cdot \frac{x-1}{5x} \quad \text{Multiply across}$$

$$\frac{x-1}{5x} \quad \text{Our Solution}$$

We can combine multiplying and dividing of fractions into one problem as shown below. To solve we still need to factor, and we use the reciprocal of the divided fraction.

Example 6.

$$\frac{a^2+7a+10}{a^2+6a+5} \cdot \frac{a+1}{a^2+4a+4} \div \frac{a-1}{a+2} \quad \text{Factor each expression}$$

$$\frac{(a+5)(a+2)}{(a+5)(a+1)} \cdot \frac{(a+1)}{(a+2)(a+2)} \div \frac{(a-1)}{(a+2)} \quad \text{Reciprocal of last fraction}$$

$$\frac{(a+5)(a+2)}{(a+5)(a+1)} \cdot \frac{(a+1)}{(a+2)(a+2)} \cdot \frac{(a+2)}{(a-1)} \quad \text{Divide out common factors}$$

$(a+2), (a+2), (a+1), (a+5)$

$$\frac{1}{a-1} \quad \text{Our Solution}$$

World View Note: Indian mathematician Aryabhata, in the 6th century, published a work which included the rational expression $\frac{n(n+1)(n+2)}{6}$ for the sum of the first n squares $(1^1+2^2+3^2+....+n^2)$

7.2 Practice - Multiply and Divide

Simplify each expression.

1) $\frac{8x^2}{9} \cdot \frac{9}{2}$

2) $\frac{8x}{3x} \div \frac{4}{7}$

3) $\frac{9n}{2n} \cdot \frac{7}{5n}$

4) $\frac{9m}{5m^2} \cdot \frac{7}{2}$

5) $\frac{5x^2}{4} \cdot \frac{6}{5}$

6) $\frac{10p}{5} \div \frac{8}{10}$

7) $\frac{7\,(m-6)}{m-6} \cdot \frac{5m(7m-5)}{7(7m-5)}$

8) $\frac{7}{10(n+3)} \div \frac{n-2}{(n+3)(n-2)}$

9) $\frac{7r}{7r(r+10)} \div \frac{r-6}{(r-6)^2}$

10) $\frac{6x(x+4)}{x-3} \cdot \frac{(x-3)(x-6)}{6x(x-6)}$

11) $\frac{25n+25}{5} \cdot \frac{4}{30n+30}$

12) $\frac{9}{b^2-b-12} \div \frac{b-5}{b^2-b-12}$

13) $\frac{x-10}{35x+21} \div \frac{7}{35x+21}$

14) $\frac{v-1}{4} \cdot \frac{4}{v^2-11v+10}$

15) $\frac{x^2-6x-7}{x+5} \cdot \frac{x+5}{x-7}$

16) $\frac{1}{a-6} \cdot \frac{8a+80}{8}$

17) $\frac{8k}{24k^2-40k} \div \frac{1}{15k-25}$

18) $\frac{p-8}{p^2-12p+32} \div \frac{1}{p-10}$

19) $(n-8) \cdot \frac{6}{10n-80}$

20) $\frac{x^2-7x+10}{x-2} \cdot \frac{x+10}{x^2-x-20}$

21) $\frac{4m+36}{m+9} \cdot \frac{m-5}{5m^2}$

22) $\frac{2r}{r+6} \div \frac{2r}{7r+42}$

23) $\frac{3x-6}{12x-24}(x+3)$

24) $\frac{2n^2-12n-54}{n+7} \div (2n+6)$

25) $\frac{b+2}{40b^2-24b}(5b-3)$

26) $\frac{21v^2+16v-16}{3v+4} \div \frac{35v-20}{v-9}$

27) $\frac{n-7}{6n-12} \cdot \frac{12-6n}{n^2-13n+42}$

28) $\frac{x^2+11x+24}{6x^3+18x^2} \cdot \frac{6x^3+6x^2}{x^2+5x-24}$

29) $\frac{27a+36}{9a+63} \div \frac{6a+8}{2}$

30) $\frac{k-7}{k^2-k-12} \cdot \frac{7k^2-28k}{8k^2-56k}$

31) $\frac{x^2-12x+32}{x^2-6x-16} \cdot \frac{7x^2+14x}{7x^2+21x}$

32) $\frac{9x^3+54x^2}{x^2+5x-14} \cdot \frac{x^2+5x-14}{10x^2}$

33) $(10m^2+100m) \cdot \frac{18m^3-36m^2}{20m^2-40m}$

34) $\frac{n-7}{n^2-2n-35} \div \frac{9n+54}{10n+50}$

35) $\frac{7p^2+25p+12}{6p+48} \cdot \frac{3p-8}{21p^2-44p-32}$

36) $\frac{7x^2-66x+80}{49x^2+7x-72} \div \frac{7x^2+39x-70}{49x^2+7x-72}$

37) $\frac{10b^2}{30b+20} \cdot \frac{30b+20}{2b^2+10b}$

38) $\frac{35n^2-12n-32}{49n^2-91n+40} \cdot \frac{7n^2+16n-15}{5n+4}$

39) $\frac{7r^2-53r-24}{7r+2} \div \frac{49r+21}{49r+14}$

40) $\frac{12x+24}{10x^2+34x+28} \cdot \frac{15x+21}{5}$

41) $\frac{x^2-1}{2x-4} \cdot \frac{x^2-4}{x^2-x-2} \div \frac{x^2+x-2}{3x-6}$

42) $\frac{a^3+b^3}{a^2+3ab+2b^2} \cdot \frac{3a-6b}{3a^2-3ab+3b^2} \div \frac{a^2-4b^2}{a+2b}$

43) $\frac{x^2+3x+9}{x^2+x-12} \cdot \frac{x^2+2x-8}{x^3-27} \div \frac{x^2-4}{x^2-6x+9}$

44) $\frac{x^2+3x-10}{x^2+6x+5} \cdot \frac{2x^2-x-3}{2x^2+x-6} \div \frac{8x+20}{6x+15}$

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Answers - Multipy and Divide

1) $4x^2$

2) $\frac{14}{3}$

3) $\frac{63}{10n}$

4) $\frac{63}{10m}$

5) $\frac{3x^2}{2}$

6) $\frac{5p}{2}$

7) $5m$

8) $\frac{7}{10}$

9) $\frac{r-6}{r+10}$

10) $x+4$

11) $\frac{2}{3}$

12) $\frac{9}{b-5}$

13) $\frac{x-10}{7}$

14) $\frac{1}{v-10}$

15) $x+1$

16) $\frac{a+10}{a-6}$

17) 5

18) $\frac{p-10}{p-4}$

19) $\frac{3}{5}$

20) $\frac{x+10}{x+4}$

21) $\frac{4(m-5)}{5m^2}$

22) 7

23) $\frac{x+3}{4}$

24) $\frac{n-9}{n+7}$

25) $\frac{b+2}{8b}$

26) $\frac{v-9}{5}$

27) $-\frac{1}{n-6}$

28) $\frac{x+1}{x-3}$

29) $\frac{1}{a+7}$

30) $\frac{7}{8(k+3)}$

31) $\frac{x-4}{x+3}$

32) $\frac{9(x+6)}{10}$

33) $9m^2(m+10)$

34) $\frac{10}{9(n+6)}$

35) $\frac{p+3}{6(p+8)}$

36) $\frac{x-8}{x+7}$

37) $\frac{5b}{b+5}$

38) $n+3$

39) $r-8$

40) $\frac{18}{5}$

41) $\frac{3}{2}$

42) $\frac{1}{a+2b}$

43) $\frac{1}{x+2}$

44) $\frac{3(x-2)}{4(x+2)}$